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STRESS DISTRIBUTION IN A STRIP WITH A THIN ELASTIC INCLUSION

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The problem of the stress distribution in a strip with a rectilinear arbitrarily oriented thin-walled elastic inclusion of finite length is considered. The problem is reduced to finding the solution of a system of four singular integral equations by using a Fourier integral transform. Numerical values are presented for the stress intensity coefficients on the endfaces of the elastic inclusion.

The state of stress of a piecewise-homogeneous plane with a thin-walled elastic inclusion of finite length was investigated in [1, 2]. A solution of the corresponding periodic problem is obtained in [3].

1. The elastic equilibrium of a homogeneous isotropic strip of width H with an arbitrarily located thin-walled inclusion of width $2h$ localized along the segment $[a, b]$ within the strip (Fig. 1) is considered.

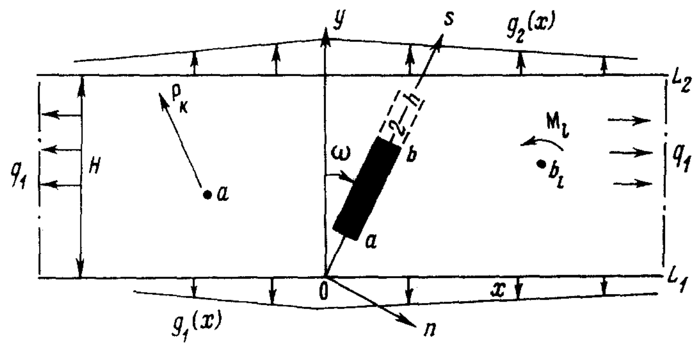

Fig. 1

Besides the Cartesian xoy coordinate system, let us introduce a son coordinate system obtained by rotating the xoy system through an angle ω. Let $\{\sigma_{xx}, \sigma_{yy}, \sigma_{xy}\}$ and $\{\sigma_{ss}, \sigma_{nn}, \sigma_{sn}\}$ be the stress tensor components defined in the xoy and son coordinate systems, respectively.

The strip is subjected to a homogeneous stress field at infinity $\sigma_{xx}(\infty, y) - i\sigma_{xy}(\infty, y) = q_1 - iq_2$, forces distributed along the side faces of the strip $\sigma_{yy} - i\sigma_{xy} = g_j(x)$ on L_j $(j = 1, 2)$, where $\lim_{x\to\infty} g_j(x) = q_3 - iq_2$, as well as systems of concentrated forces $\mathbf{P}_k$ and moments $\mathbf{M}_l$ applied at the internal points of the strip a_k and b_l, respectively $(k = 1, \ldots, n;\ l = 1, \ldots, m)$. The inclusion is free of external loads. Let us determine the stress distribution in the strip in the neighborhood of the inclusion.

The assumption regarding the small thickness of the inclusion permits the consider from that its presence can be modeled by a stress jump and derivatives of the displacement in the homogeneous strip on a segment coincident with the middle line of the real inclusion, i.e.,

$$[\sigma_{nn}(s,+0)-i\sigma_{sn}(s,+0)]-[\sigma_{nn}(s,-0)-i\sigma_{sn}(s,-0)]= \tag{1.1}$$

$$f_1(s)-if_2(s)$$

$$[u'(s,+0)+iv'(s,+0)]-[u'(s,-0)+iv'(s,-0)]=f_3(s)+if_4(s)$$

$$f_j(s)=0,\ s\,\overline{\in}\,[a,b];\ u'=\partial u/\partial s,\ v'=\partial v/\partial s$$

Here u, v are the displacements; the superscripts plus and minus characterize the stress and displacement field components of the strip on the upper and lower edges of the inclusion, respectively.

We have four conditions [3] for the interaction between the thin-walled elastic inclusion and the surrounding medium to determine the unknown functions $f_j(s)$ $(j = 1, \ldots, 4)$:

$$\partial[u(s,-0)+u(s,+0)]/\partial s=2k_{10}\sigma_s-k_{20}[\sigma_{nn}(s,-0)+ \tag{1.2}$$

$$\sigma_{nn}(s,+0)]$$

$$[v(s,-0)-v(s,+0)]/h=-2k_{20}\sigma_s+k_{10}[\sigma_{nn}(s,-0)+\sigma_{nn}(s,+0)]$$

$$\partial[v(s,-0)+v(s,+0)]/\partial s+[u(s,-0)+u(s,+0)]/h=$$

$$[\sigma_{sn}(s,-0)+\sigma_{sn}(s,+0)]/\mu_0$$

$$\partial[u(s,-0)-u(s,+0)]/\partial s=k_{30}[\sigma_{nn}(s,+0)-\sigma_{nn}(s,-0)]$$

$$\sigma_s=N_a-\frac{1}{2h}\int_a^s[\sigma_{sn}(t,-0)-\sigma_{sn}(t,+0)]\,dt$$

$$k_{10}=(1+\varkappa_0)/(8\mu_0),\quad k_{20}=(3-\varkappa_0)/(8\mu_0)$$

$$k_{30}=[(k_{20})^2-(k_{10})^2]/k_{10},\quad \mu_j=E_j/[2(1+\nu_j)]$$

Here for plane strain $\varkappa_j = 3-4\nu_j$, for the generalized plane stress $\varkappa_j = (3-\nu_j)/(1+\nu_j)$; E_j, ν_j are, respectively, the elastic modulus and Poisson's ratio of the material of the inclusion $(j = 0)$ and the matrix $(j = 1)$, and N_a is the normal stress on the endface $s = a$ of the inclusion.

2. Let us represent the solution of the problem S in the form of the sum of solutions S° of the corresponding problem without an inclusion, and S^* of the problem of the elastic equilibrium of a strip with a mathematical slit along the segment $[a, b]$ under the conditions

$$\begin{aligned}
&\sigma_{nn}^*(s,\pm 0)=\sigma_{nn}(s,\pm 0)-\sigma_{nn}^\circ(s)\\
&\sigma_{sn}^*(s,\pm 0)=\sigma_{sn}(s,\pm 0)-\sigma_{sn}^\circ(s)\\
&u^{*\prime}(s,\pm 0)=u'(s,\pm 0)-u^{\circ\prime}(s)\\
&v^*(s,\pm 0)=v'(s,\pm 0)-v^{\circ\prime}(s)\\
&\sigma_{yy}^*(x,0)=\sigma_{yy}^*(x,H)=0,\ \sigma_{xy}^*(x,0)=\sigma_{xy}^*(x,H)=0
\end{aligned} \tag{2.1}$$

We note that the stress jump and the derivatives of the displacement on the edges of the segment equal the corresponding jumps on the edges of the inclusion.

We seek the stress function $\Phi^*(s, n)$ in the form of the sum of two functions

$$\Phi^*(s, n) = \Phi^1(s, n) + \Phi^2(x, y) \tag{2.2}$$

Here $\Phi^1(s, n)$ defines the solution of S^1 of the problem for an infinite plane with jumps $f_j(s)$ $(j = 1, \ldots, 4)$ in the stresses and derivatives of the displacement at the appropriate segment, and $\Phi^2(x, y)$ is the stress function for a continuous inclusion subjected at the side faces to forces equal in magnitude but opposite in sign to the stresses defined by the function $\Phi^1(s, n)$ on L_j $(j = 1, 2)$.

According to [4], the integral representation of the stress function $\Phi^1(s, n)$ has the form

$$\Phi^1(s, n) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_1^{\pm}(\eta, n)\, e^{-i\eta s}\, d\eta \tag{2.3}$$

$$G_1^+(\eta, n) = [A_1(\eta) + n\,|\eta|\,A_2(\eta)]\, e^{-|\eta| n}, \quad n > 0$$

$$G_1^-(\eta, n) = [A_3(\eta) + n\,|\eta|\,A_4(\eta)]\, e^{|\eta| n}, \quad n < 0$$

Here $A_j(\eta)$ $(j = 1, \ldots, 4)$ are complex functions.

Using the expressions for the stress tensor components and the derivatives of the displacement vector in terms of the function $\Phi^1(s, n)$, satisfying the conditions (1.1), and applying the inverse Fourier transform, we obtain an algebraic system of equations to determine $A_j(\eta)$ $(j = 1, \ldots, 4)$ in the form

$$-\eta^2[A_1(\eta) - A_3(\eta)] = F_1^*(\eta)$$

$$i\eta\,|\eta|\,[-A_1(\eta) + A_2(\eta) - A_3(\eta) - A_4(\eta)] = F_2^*(\eta)$$

$$\eta^2[A_1(\eta) - 2A_2(\eta) - A_3(\eta) - 2A_4(\eta)] = [F_3^*(\eta) + k_2 F_1^*(\eta)] / k_1$$

$$i\eta\,|\eta|\,[A_1(\eta) + 3A_2(\eta) - A_3(\eta) - 3A_4(\eta)] = [F_4^*(\eta) + k_3 F_2^*(\eta)] / k_1$$

$$F_j^*(\eta) = \int_a^b f_j(t)\, e^{i\eta t}\, dt, \quad k_1 = \frac{(1 + \varkappa_1)}{8\mu_1}$$

$$k_2 = \frac{(3 - \varkappa_1)}{8\mu_1}, \quad k_3 = \frac{(5 + \varkappa_1)}{8\mu_1}$$

The expressions for the characteristics of the stress-strain state in the problem S^1 in terms of the unknown jumps $f_j(s)$ $(j = 1, \ldots, 4)$ have the form

$$\{\sigma_{mn}^1, u^{1\prime}, v^{1\prime}\} = \sum_{j=1}^{4} \frac{1}{\pi}\int_a^{b} \{P_{mn}(t - s, n),\ P_u(t - s, n),\ P_v(t - s, n)\}\, f_j(t)\, dt$$

Here P_{mn}, P_u, P_v are functions having both a singular and a regular part in the limit case $n \to 0$.

The function $\Phi^2(x, y)$ has the representation [4]

$$\Phi^2(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G_2(\xi, y) e^{-i\xi x} d\xi$$

$$G_2(\xi, y) = [B_1(\xi) + \xi y B_2(\xi)] e^{-\xi y} + [B_3(\xi) + \xi y B_4(\xi)] e^{\xi y}$$

Here $B_j(\xi)$ $(j = 1, \ldots, 4)$ are unknown complex functions

Introducing the notation $\alpha = \cos\omega$, $\beta = \sin\omega$, we obtain from (2.1) — (2.3)

$$-\frac{\partial^2\Phi^2}{\partial x^2} = \alpha^2 \frac{\partial^2\Phi^1}{\partial n^2} + \beta^2 \frac{\partial^2\Phi^1}{\partial s^2} + 2\alpha\beta \frac{\partial^2\Phi^1}{\partial s\, \partial n}$$

$$-\frac{\partial^2\Phi^2}{\partial x\, \partial y} = \alpha\beta\left(\frac{\partial^2\Phi^1}{\partial s^2} - \frac{\partial^2\Phi^1}{\partial n^2}\right) + (\alpha^2 - \beta^2)\frac{\partial^2\Phi^1}{\partial s\partial n} \quad \text{on } L_j \quad (j = 1, 2)$$

Hence, an algebraic fourth-order system follows for the determination of the unknowns $B_j(\xi)$ $(j = 1, \ldots, 4)$. Solving this system we represent the solution of S^2 in the form

$$\{\sigma^2_{mn}, u^{2\prime}, v^{2\prime}\} = \sum_{j=1}^{4} \frac{1}{\pi} \int_a^b \{Q_{mn}(t, s, n), \quad Q_u(t, s, n), \quad Q_v(t, s, n)\} f_j(t)\, dt$$

Here Q_{mn}, Q_u, Q_v are regular functions.

We determine the stress-strain state of the body under consideration at any of its points by means of the solutions of S°, S^1, S^2 :

$$\begin{aligned}
\sigma_{nn}(s, n) &= \sigma_{nn}{}^\circ(s, n) + \sigma_{nn}{}^1(s, n) + \sigma_{nn}{}^2(s, n) \\
\sigma_{sn}(s, n) &= \sigma_{sn}{}^\circ(s, n) + \sigma_{sn}{}^1(s, n) + \sigma_{sn}{}^2(s, n) \\
u'(s, n) &= u^{\circ\prime}(s, n) + u^{1\prime}(s, n) + u^{2\prime}(s, n) \\
v'(s, n) &= v^{\circ\prime}(s, n) + v^{1\prime}(s, n) + v^{2\prime}(s, n)
\end{aligned} \tag{2.4}$$

Passing to the limit in (2.4) as $n \to +0$, we obtain the following expressions for the stress and the derivatives of the displacement on the upper edge of the inclusion (values of the corresponding quantities on the lower edge of the inclusion are determined from (1.1) and (2.5)):

$$\begin{aligned}
\sigma_{nn}(s, +0) &= \sigma_{nn}{}^\circ(s) + {}^1/_2 f_1(s) - m_1 t_2(s) - n_1 t_4(s) + k_1(s) \\
\sigma_{sn}(s, +0) &= \sigma_{sn}{}^\circ(s) + {}^1/_2 f_2(s) + m_1 t_1(s) - n_1 t_3(s) + k_2(s) \\
u'(s, +0) &= u^{\circ\prime}(s) + {}^1/_2 f_3(s) + m_2 t_2(s) + m_1 t_4(s) + k_3(s) \\
v'(s, +0) &= v^{\circ\prime}(s) + {}^1/_2 f_4(s) + m_2 t_1(s) - m_1 t_3(s) + k_4(s)
\end{aligned} \tag{2.5}$$

$$t_i(s) = \frac{1}{\pi} \int_a^b \frac{f_j(t)\, dt}{t - s}, \quad k_i(s) = \sum_{j=1}^{4} \int_a^b K_{ij}(s, t) f_j(t)\, dt \quad (i = 1, \ldots, 4)$$

$$K_{1j}(s, t) = S_{1j}(\beta s, \alpha s, t) + S_{2j}(\beta s, \alpha s, t)\cos 2\omega - S_{3j}(\beta s, \alpha s, t)\sin 2\omega$$

$$K_{2j}(s, t) = S_{2j}(\beta s, \alpha s, t)\sin 2\omega + S_{3j}(\beta s, \alpha s, t)\cos 2\omega$$

$$K_{3j}(s, t) = -4 [m_1 S_{1j}(\beta s, \alpha s, t) - n_3 [S_{2j}(\beta s, \alpha s, t) \cos 2\omega - S_{3j}(\beta s, \alpha s, t) \sin 2\omega]]$$
$$K_{4j}(s, t) = -4\alpha [m_1 S_{1j}(\beta s, \alpha s, t) - S_{2j}(\beta s, \alpha s, t)] - \beta [k_3 S_{3j}(\beta s, \alpha s, t) - k_1 S_{4j}(\beta s, \alpha s, t)]$$

$$S_{ij}(x, y, t) = \frac{1}{\pi} \int_0^\infty \left[M_{ij}(x, y, t, \xi) - M_{ij}\left(\beta \frac{H}{\alpha} - x, \alpha \frac{H}{\alpha} - y, \frac{H}{\alpha} - t, \xi\right) \right] d\xi \tag{2.6}$$

$$M_{1j}(x, y, t, \xi) = (\Delta_{1j} H_1^+ - \Delta_{2j} H_1^-) / \Delta_0 \tag{2.7}$$
$$M_{2j}(x, y, t, \xi) = (\Delta_{1j} H_2^+ - \Delta_{2j} H_2^-) / (2\Delta_0) \quad (j = 1, 2)$$
$$M_{ij}(x, y, t, \xi) = (-1)^j [\Delta_{3k} H_i^+ + \Delta_{4k} H_i^-] / (2\Delta_0)$$
$$i = 3, 4;\ j = 1, 2\ (j = 1,\ k = 2;\ j = 2,\ k = 1)$$
$$\{\Delta_{1i}, \Delta_{2i}, \Delta_{3i}, \Delta_{4i}\} = \{\Delta_1, \Delta_3 - \Delta_1, -\Delta_3\} \varphi_+ + \{\Delta_2, \Delta_4, -\Delta_2, -\Delta_4\} \varphi_-$$
$$i = 1,\ \varphi_+ = n_2 \sin y_+,\ \varphi_- = n_3 \sin y_-$$
$$i = 2,\ \varphi_+ = -n_2 \cos y_+,\ \varphi_- = n_3 \cos y_-$$
$$y_+ = \xi(\beta t - x) + \omega,\ y_- = \xi(\beta t - x) - \omega$$
$$\Delta_0(\xi) = (e^{\xi H} - e^{-\xi H})^2 - \xi^2 H^2,\ \Delta_1(\xi) = e^{2\xi H} - 1$$
$$\Delta_2(\xi, t) = 2\xi H + 2\alpha t \xi \Delta_1(\xi),\ \Delta_3(\xi) = 2\xi H$$
$$\Delta_4(\xi, t) = 1 - e^{-2\xi H} + 2\alpha t \xi \Delta_3(\xi)$$
$$H_1^\pm(y, t, \xi) = e^{-\xi(\alpha t \pm y)}$$
$$H_i^\pm(y, t, \xi) = H_1^\pm(y, t, \xi) H_{i-1,1}^\mp(y, \xi) \quad (i = 2, 3, 4)$$
$$H_{j,1}^\pm = r_j \pm 2\xi(H - y) + (-1)^{j-1} \pm \exp[\mp 2\xi(H - y)]$$
$$j = 1, 2, 3,\ r_1 = r_2 = 1,\ r_3 = 5$$
$$m_1 = (k_3 / k_1 - 3) / 4,\quad m_2 = (k_1 + k_3 - 4 k_2 m_1) / 4$$
$$n_1 = {}^1/_4\, k_1,\ n_2 = (k_2 / k_1 - 3) / 4,\ n_3 = (k_2/k_1 + 1)/4$$

The functions $M_{ij}(x, y, t, \xi)$ $(i = 1, \ldots, 4)$ are determined by means of (2.7) for $\varphi_+ = \pm \sin y_+$, $\varphi_- = \pm \sin y_-$ (the first sign corresponds to $j = 3$ and the second to $j = 4$).

3. The relationships (2.5) and the interaction conditions between the thin-walled elastic inclusion and the matrix (1.2) result in a system of singular integral equations

$$t_2(s) + \lambda_{11} t_4(s) - \lambda_1 \int_a^s f_2(t)\, dt + R_1(s) = F_1(s) \tag{3.1}$$

$$t_2(s) + \lambda_{21} t_1(s) - \lambda_2 \int_a^s f_4(t)\, dt + R_2(s) = F_2(s)$$

$$t_4(s) + \lambda_{31} t_2(s) + \int_a^s [\lambda_3 f_2(t) + \lambda_4 f_4(t)]\, dt + R_3(s) = F_3(s)$$

$$
\begin{aligned}
&f_3(s) = -k_{30} f_1(s), \quad s \in [a, b] \\
&R_1(s) = \lambda_{12} k_1(s) - \lambda_{13} k_3(s), \quad R_2(s) = \lambda_{22} k_2(s) + \lambda_{23} k_4(s) \\
&R_3(s) = \lambda_{32} k_1(s), \quad F_1(s) = [k_{10} N_a - u^{\circ\prime}(s) - k_{20} \sigma_{nn}^{\circ}(s)] / \Lambda_1 \\
&F_2(s) = \mu_0 [v^{\circ\prime}(s) - \sigma_{ns}^{\circ}(s) / \mu_0 - c_a / (2h)] / \Lambda_2 \\
&F_3(s) = [\sigma_{nn}^{\circ}(s) + d_a / (2hk_{10}) - \nu_0 N_a] / \Lambda_3 \\
&\lambda_{11} = (4m_1 - k_{20} / k_1) / (4\Lambda_1), \quad \lambda_{12} = k_{20} / \Lambda_1, \quad \lambda_{13} = 1 / \Lambda_1 \\
&\lambda_{21} = (\mu_0 m_2 - m_1)/\Lambda_2, \quad \lambda_{22} = 1/\Lambda_2, \quad \lambda_{23} = \mu_0/\Lambda_2 \\
&\lambda_{31} = m_1 / \Lambda_3 \\
&\lambda_{32} = 1 / \Lambda_3, \quad \lambda_1 = k_{10} / (2h\Lambda_1), \quad \lambda_2 = \mu_0 / (2h\Lambda_2) \\
&\lambda_3 = \nu_0 / (2h\Lambda_3), \quad \lambda_4 = -1 / (2hk_{10}\Lambda_3) \\
&\Lambda_1 = m_2 - k_{20} m_1, \quad \Lambda_2 = (n_1 - m_1\mu_0), \quad \Lambda_3 = n_1
\end{aligned}
$$

The required functions satisfy the additional conditions (3.2)

$$
\begin{aligned}
&\int_a^b f_j(t)\,dt = A^j \quad (j = 1, \ldots, 4) \\
&A^1 = 0, \; A^2 = 2h(N_b - N_a), \; A^3 = c_b - c_a, \; A^4 = d_b - d_a
\end{aligned}
\tag{3.2}
$$

The normal stresses on the inclusion endfaces, as well as the displacements c_s, d_s $(s = a, b)$ of the lower point of the endface of the inclusion relative to its upper points are evaluated by means of formulas in [3].

In the case of an absolutely rigid inclusion $E_0 \to \infty$, the system (3.1) is converted to the form

$$
\begin{aligned}
&m_2 t_1(s) - k_3(s) = -v^{\circ\prime}(s), \quad m_2 t_2(s) - k_4(s) = -u^{\circ\prime}(s) \\
&f_3(s) = f_4(s) = 0, \quad s \in [a, b]
\end{aligned}
\tag{3.3}
$$

When $E_0 \to 0$, we obtain a system of singular integral equations for a crack in the strip

$$
\begin{aligned}
&n_1 t_4(s) + k_1(s) = \sigma_{ss}^{\circ}(s), \quad n_1 t_3(s) + k_2(s) = \sigma_{ns}^{\circ}(s) \\
&f_1(s) = f_2(s) = 0, \quad s \in [a, b]
\end{aligned}
\tag{3.4}
$$

Equations (3.4) agree with the results obtained in [5]

If H is allowed to tend to infinity in the integral equations (3.1), then we obtain the solution of the problem for a half-plane with an inclusion. In this case the Fredholm kernels $S_{ij}(x, y, t, \xi)$ (2.6) are evaluated in closed form.

4. We seek the solution of the system of singular integral equations (3.1) in the form

$$
\begin{aligned}
&f_j(\xi) = \left[A_0^j + \sum_{n=1}^{\infty} A_n^j T_n(\xi)\right] \Big| \sqrt{1-\xi^2} \quad (j = 1, \ldots, 4) \\
&\xi = (2s - a - b) / (b - a)
\end{aligned}
\tag{4.1}
$$

Here $T_n(\xi)$ are Chebyshev (Tschebyscheff) polynomials of the first kind. Substituting

(4.1) into conditions (3.2) and integrating, we obtain $A_0^j = A^j / [\pi 2^{-1}(b-a)]$.

Substitution of the series (4.1) into the system of integral equations (3.1) and the ordinary procedure of the method of orthogonal polynomials [6,7] result in a system of algebraic equations to determine the desired expansion coefficients:

$$\frac{\pi}{2}\lambda_{11}A_k^4 + \sum_{n=1}^{\infty}\left[\left(\delta_{kn}\frac{\pi}{2} + \lambda_1 B_{n-1,k-2}\right)A_n^2 + \sum_{j=1}^{4} H_{n,k-1}^{1j}A_n^j\right] = F_{k-1}^1 \quad (4.2)$$

$$\frac{\pi}{2}\lambda_{21}A_k^1 + \sum_{n=1}^{\infty}\left[\left(\delta_{kn}\frac{\pi}{2} + \lambda_2 B_{n-1,k-2}\right)A_n^3 + \sum_{j=1}^{4} H_{n,k-1}^{2j}A_n^j\right] = F_{k-1}^2$$

$$\sum_{n=1}^{\infty}\left[\left(\delta_{kn}\frac{\pi}{2} - \lambda_4' B_{n-1,k-2}\right)A_n^4 + \left(\lambda_{31}\delta_{kn}\frac{\pi}{2} - \lambda_3' B_{n-1,k-2}\right)A_n^2 + \sum_{j=1}^{4} H_{n,k-1}^{3j}A_n^j\right] = F_{k-1}^3$$

$$A_n^3 = -k_{30}A_n^1 \quad (k = 1, 2, \ldots)$$

$$F_k^i = \Phi_k^i - \lambda_i'\delta_k A_0^{i+1} - \sum_{j=1}^{4} H_{0k}^{ij}A_0^j \quad (i = 1, 2)$$

$$F_k^3 = \Phi_k^3 + (\lambda_3' A_0^2 + \lambda_4' A_0^4)\delta_k - \sum_{j=1}^{4} H_{0k}^{3j}A_0^j$$

$$\Phi_k^p = \int_{-1}^{1} \varphi_p(\zeta)U_k(\zeta)\sqrt{1-\zeta^2}\,d\zeta$$

$$\varphi_i(\zeta) = F_i(\zeta) + A^{i+1}\lambda_i' / (b-a) \quad (i = 1, 2)$$

$$\varphi_3(\zeta) = F_3(\zeta) - (\lambda_3' A^2 + \lambda_4' A^4) / (b-a)$$

$$B_{n-1,k} = \frac{1}{n^2-k^2} - \frac{1}{n^2-(k-2)^2}\quad \lambda_i' = \lambda_i\frac{(b-a)}{2}$$

$$H_{nk}^{ij} = \int_{-1}^{1} U_k(\zeta)\sqrt{1-\zeta^2}\int_{-1}^{1} K_{ij}(\zeta,\tau)T_n(\tau)\frac{d\tau}{\sqrt{1-\tau^2}}\,d\zeta$$

Here $U_k(\zeta)$ are Chebyshev polynomials of the second kind, $\delta_k = 0$ for even k and $\delta_k = B_{-1,k}$ for odd k.

As an illustration, let us consider the case of equlibrium of a strip with a thin-walled elastic inclusion of length $2a$ located on the line of symmetry of a strip subjected to constant normal forces on its faces $\sigma_{yy}(x, 0) = \sigma_{yy}(x, H) = q$.

In this case $A^j = 0$, $\varphi_p(\zeta) = F_p(\zeta)$, $F_2(\zeta) = 0$ $(j = 1, \ldots, 4;\ p = 1, 2, 3)$, and $F_1(\zeta)$ and $F_2(\zeta)$ are odd functions. There then follows from the system (4.2)

$$f_j(\xi) = \left[\sum_{n=1}^{\infty} A_{2n-1}^j T_{2n-1}(\xi)\right] \Big/ \sqrt{1-\xi^2} \quad (j = 2, 4) \quad (4.3)$$

$$f_1(s) = f_3(s) = 0,\quad A_k^1 = A_k^3 = 0,\quad A_{2k}^2 = A_{2k}^4 = 0 \quad (k = 1, 2, \ldots)$$

The stress intensity coefficients at the right endface of the inclusion are determined from the formulas

$$\{k_{-1}^{\pm}(a),\, k_{-2}^{\pm}(a)\} = \lim_{s\to a-0} [\sqrt{a-s}\,\{\sigma_{nn}(s, \pm 0),\ \sigma_{sn}(s, \pm 0)\}]$$

$$\{k_{+1}(a),\ k_{+2}(a)\} = \lim_{s\to a+0} [\sqrt{s-a}\,\{\sigma_{nn}(s, 0),\ \sigma_{sn}(s, 0)\}]$$

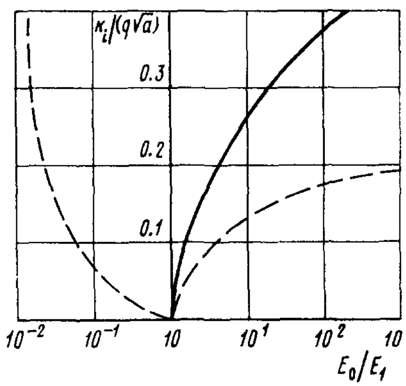

Fig. 2

In the case under consideration, the system of singular integral equations is solved numerically. To achieve a 2% accuracy in the calculations for different values of the relative width of the strip H/a from 15 to 40 terms in the expansion (4.3) are required. The convergence of the calculation process was investigated by comparing the last and its preceding approximations. Results of a calculation of the dependence of the stress intensity coefficients k_1 and k_2 on the relative rigidity of the inclusion E_0/E_1 are represented in Fig. 2. The solid line corresponds to k_2 and the dashed one to k_1.

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